Full Characterization of Linear Acoustic Networks Based on *N*-Ports and S Parameters*

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A new method is introduced to characterize (segments of) a closed acoustic system. Because it is a full characterization, it is possible to calculate all reflection and transmission effects of a composite system, even if no model is available of the separate segments. A measuring instrument used for this characterization, which is an acoustic network analyzer, is described. The parameterization fit well with systems in which traveling waves propagate. For this reason the method is commonly used by electrical engineers to analyze microwave circuits (used in radar and in satellite links).

0 INTRODUCTION

A common method to calculate the relationship between the input and output of an acoustic system is to split up the system into smaller parts (Fig. 1) and create a model of each part or segment that predicts the relation between average sound pressure p and volume flow rate u. The average sound pressure and volume flow rate are the surface integral of pressure p and velocity v in every point of the aperture A of the segment,

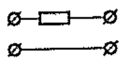
$$p = \frac{1}{A} \iint p \cdot \mathrm{d}A \quad [\mathrm{N/m^2}]$$

$$u = \iint v \cdot \mathrm{d}A \quad [\mathrm{m^3/s}] \ .$$

When the mathematical relation between p and u is known for each segment, it is possible to calculate the dynamic properties of the complete composite system [1], [2].

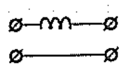
In the analogy of electronic networks the average sound pressure p can be represented by electrical voltage and the volume flow rate u by electrical current. In case where special mathematical relations hold between sound pressure and volume flow rate, the segment can be represented by ideal electrical components such as resistors, capacitors, and inductors [1], [2]:

Resistance:



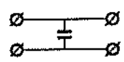
$$\text{if } u_1 = u_2 = u \text{ and } \frac{\Delta p}{u} = \text{constant}$$

Inductance:



$$\text{if } u_1 = u_2 = u \text{ and } \frac{\Delta p}{\mathrm{d}u/\mathrm{d}t} = \text{constant}$$

Capacitance:



$$\text{if } p_1 = p_2 = p \text{ and } \frac{\mathrm{d}p/\mathrm{d}t}{\Delta u} = \text{constant} \ .$$

Using this analogy, the properties of an acoustic network can be calculated with the same mathematical tools as electric networks [3].

This lumped-element method is very useful when the closed system can easily be split into segments and all individual segments can be represented by simple models. However, the usefulness declines when infinite segments are required to represent the system. Espe-

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cially when the volume of these segments cannot be neglected with respect to the acoustic wavelength, delay time will become an important factor. The use of a two-port description in combination with ideal lumped circuit elements will solve this problem.

1 FULL TWO-PORT DESCRIPTION OF AN ACOUSTIC NETWORK

The relation between the input and output of lumped circuit elements, such as resistors, capacitors, or inductors, is defined by first-order differential equations. When acoustic systems are built up with segments which can be modeled by these lumped circuit elements, the dynamic properties of the complete system can also be expressed in terms of first-order differential equations. In the special case of harmonic sound waves, these differential equations can be simplified into linear equations by using complex transformations. The sound waves are then represented by complex numbers, where the magnitude and the phase of each phasor represent the amplitude and the phase of the sound wave, respectively.

If (p_1, u_1) and (p_2, u_2) are the complex transformed values of sound pressure and volume flow rate at the input and the output, this relation is

$$u_1 = y_{11}p_1 + y_{12}p_2$$

$$u_2 = y_{21}p_1 + y_{22}p_2$$

where $[y_{11}, y_{21}, y_{12}, y_{22}]$ are complex proportional constants which are different for each frequency and unique for a given linear acoustic system.

The same relation holds for individual segments, even when the segment must be modeled with an infinite number of (infinitely small) linear lumped circuit elements. Because these equations give a full description of the segment properties given the frequency, the segment can be replaced by a black box described by these four complex numbers. Such a black box is called a two-port and the coefficients $[y_{11}, y_{21}, y_{12}, y_{22}]$ are called Y parameters (Fig. 2).

This two-port description is not restricted to lumped segments of small volume, but applies also for large segments with respect to the acoustic wavelength. For this reason this method is commonly used by electrical engineers dealing with microwave technology.

For acoustic systems with more inputs and outputs, such as cubic cells with six junction planes, the same type of equations can be used, but with additional Y parameters.

Summarizing, the use of a full two-port representation of closed linear acoustic segments has numerous advantages with respect to models based on lumped circuit elements:

1) The description remains valid for large segments with respect to the acoustic wavelength.

2) The dynamic properties of simple acoustic systems, such as a cascade of several two-ports, can be calculated manually. This is shown in Sec. 5.

3) The dynamic properties of complicated acoustic systems with many N-ports combined with lumped circuit elements can be analyzed with commerically available simulation softwave developed for microwave circuits.

4) Because two-ports are limited by well-defined reference planes, the phase relation between input and output is well defined, too.

2 TWO-PORT DESCRIPTION BASED ON S PARAMETERS

There are many ways to rearrange the linear relations, each with its own merits. However, they are all suitable to analyze the dynamic properties of linear acoustic systems built up with an arbitrary combination of acoustic N-ports. Regardless of the form of representation used, each linear two-port is fully characterized by four complex numbers at a given frequency.

Using incident and reflected waves instead of average pressure and volume flow rate (p, u), the N-port de-

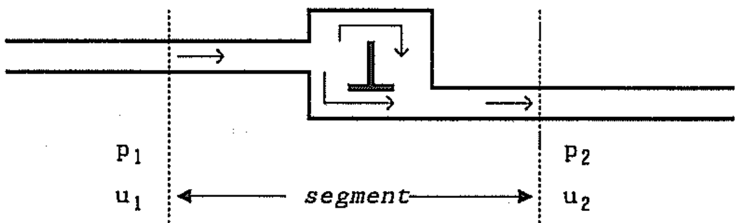


Fig. 1. Example of a segment as part of a closed acoustic system. p_1, u_1 and p_2, u_2 are average sound pressure and volume flow rate at the segment input and output, respectively.

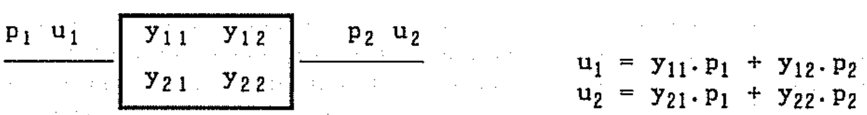


Fig. 2. Two-port description of (part of a) closed linear acoustic system based on Y parameters.

scription based on Y parameters can be transformed into a variant associated with (acoustic) plane waves [4], [5].

The (complex) value ψ of a plane acoustic wave is related to sound pressure and velocity as follows. When p^+ and u^+ represent the average sound pressure and the average velocity of the incident wave and p^- and u^- that of the reflected wave, the pressure and velocity in the junction plane of an acoustic segment are

$$p = p^+ + p^-$$

$$u = u^+ - u^- .$$

At every point of the plane wave the values of p^+ and v^+ are proportional to each other. This proportional constant is called the characteristic impedance and equals $z_0 = \rho c$ [N · s/m³], where ρ is the density and c = the velocity of sound. The relation between p^+ and u^+ is called the specific impedance of the aperture A and is equal to $\mathbf{Z}_0 = z_0/A$ [N · s/m⁵]. This relation remains valid even when the local pressure p and the local velocity v are position-dependent over the aperture A,

$$p^+ = \mathbf{Z}_0 u^+$$

$$p^- = \mathbf{Z}_0 u^- .$$

Then

$$p^+ = \frac{p + \mathbf{Z}_0 u}{2} \quad [\mathrm{N/m^2}]$$

$$p^- = \frac{p - \mathbf{Z}_0 u}{2} \quad [\mathrm{N/m^2}]$$

$$u^+ = \frac{p/\mathbf{Z}_0 + u}{2} \quad [\mathrm{m^3/s}]$$

$$u^- = \frac{p/\mathbf{Z}_0 - u}{2} \quad [\mathrm{m^3/s}] .$$

Based on these properties, we can define the complex value (ψ^+, ψ^-) of a plane wave through an aperture as

$$\psi^+ = \sqrt{\mathbf{Z}_0}\left(\frac{p}{\mathbf{Z}_0} + u\right) \quad [\sqrt{\mathrm{W}}]$$

$$\psi^- = \sqrt{\mathbf{Z}_0}\left(\frac{p}{\mathbf{Z}_0} - u\right) \quad [\sqrt{\mathrm{W}}] .$$

Because the complex waves (ψ^+, ψ^-) are defined based on a linear relation between p and u, the two-port description of Fig. 2 can be remodeled to the linear form, illustrated in Fig. 3.

Summarizing, a two-port description with S parameters has a number of advantages with respect to a description based on Y parameters:

1) They fit well with the transmission-line approximation of the tubes guiding traveling sound waves.

2) The four complex S parameters have a physical significance and can be determined directly from measurements. This will be worked out in Sec. 3.

3) Their values will always remain finite, even when one of the Y parameters equals infinity in the same situation.

4) In case of special segments, such as symmetrical, reciprocal, or loss-free segments, their characteristics can be expressed by simple relations between S parameters. The same holds for ideal transmission lines.

3 PHYSICAL SIGNIFICANCE OF S PARAMETERS

The relation between S parameters and traveling waves can be expressed as follows:

$$\psi_1^- = s_{11}\psi_1^+ + s_{12}\psi_2^-$$

$$\psi_2^+ = s_{21}\psi_1^+ + s_{22}\psi_2^- .$$

Hence,

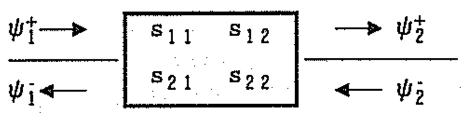
Input reflection coefficient: $s_{11} = \dfrac{\psi_1^-}{\psi_1^+}$ if $\psi_2^- = 0$

Forward transmission: $s_{21} = \dfrac{\psi_2^+}{\psi_1^+}$ if $\psi_2^- = 0$

Reverse transmission: $s_{12} = \dfrac{\psi_1^-}{\psi_2^-}$ if $\psi_1^+ = 0$

Output reflection coefficient: $s_{22} = \dfrac{\psi_2^+}{\psi_2^-}$ if $\psi_1^+ = 0$

Here s_{11} is the reflection coefficient at the input when the output is matched (perfect absorption of all incident waves); s_{21} is the transmission coefficient in the forward direction when the output is matched; s_{12} is the transmission coefficient in reverse direction when the input is matched; and s_{22} is the reflection coefficient at the



$$\psi_1^- = s_{11}\cdot\psi_1^+ + s_{12}\cdot\psi_2^-$$

$$\psi_2^+ = s_{21}\cdot\psi_1^+ + s_{22}\cdot\psi_2^-$$

Fig. 3. Two-port description of (part of a) closed linear acoustic system based on S parameters. They describe the outgoing waves (ψ_1^-, ψ_2^+) as a function of the incoming waves (ψ_1^+, ψ_2^-). Acoustic systems with more connection planes can be described the same way, but with more S parameters.

output when the input is matched.

Like electrical networks, special physical features of acoustic networks can be expressed by simple relations of its S parameters:

Reciprocal networks: $s_{21} = s_{12}$

Symmetrical networks: $s_{21} = s_{12}$

$s_{11} = s_{22}$

Loss-free networks: $|s_{11}|^2 + |s_{21}|^2 = 1$

$|s_{22}|^2 + |s_{12}|^2 = 1$

Ideal transmission lines (Z_0): $s_{21} = s_{12} = e^{-j\phi}$

$s_{11} = s_{22} = 0$.

4 MEASUREMENTS OF S PARAMETERS WITH AN ACOUSTIC NETWORK ANALYZER

In principle the S parameters of an acoustic network can be derived by modeling the network with ideal lumped circuit elements. However, this is not always feasible, especially in the case of complicated networks. On the other hand, all four complex S parameters can always be measured.

The measuring method is based on the measurement of the ratio between incident, transmitted, and reflected waves. The ratio between these waves can be reconstructed from measurements of average sound pressure and phase difference at well-defined positions. In analogy with a comparable instrument, used for the measurements of S parameters of microwave circuits, we call the measurement setup as described here an acoustic network analyzer.

The basic setup of the network analyzer is shown in Fig. 4. It consists of two acoustic waveguides with the two-port to be measured in between. The transitions from the waveguides to the two-port form the well-defined reference planes (4) and (5). In the left waveguide, a harmonic acoustic wave is excited by a loudspeaker (1). This wave propagates through the first reference plane (4) toward the measuring object. In general, a part of the wave will be reflected there, causing a standing-wave pattern in the left waveguide. With a movable microphone (3) the standing-wave ratio and the position of the pressure maximum can be measured with respect to the first reference plane (4).

To prevent multiple reflections, absorbing material (glass wool) is used to dissipate the reflected waves toward the loudspeaker. With another microphone (6), located near the second reference plane (5), the sound pressure and phase (with respect to a reference phase) at the output of the two-port will be measured. The right waveguide is also matched by absorbing material (7).

The quality of the absorption can be verified by standing-wave measurements when both waveguides are connected to each other. The length and density of the absorbing material (7) must be well chosen to perform constant sound pressure at any position along the waveguide. The same holds for adjusting the length and density of the absorbing material at the left side (2). This can be verified by using another loudspeaker, located at the first reference plane (4) in the opposite direction.

With the measuring setup described we can reconstruct the four complex S parameters from eight real ratio measurements: the first four real values with the two-port measured in the forward direction and another four identical real values with the two-port in the reverse direction. The following quantities are to be measured:

1) The standing-wave ratio SWR ($= p_{max}/p_{min}$) at the input, measured with the movable microphone (3) shifted from the position with a maximum in sound pressure to the position with a minimum in pressure. Because the distance between pressure maximum and minimum is exactly a quarter wavelength ($\lambda/4$), this measurement gives also the system constant λ.

2) The measured distance x between the first reference plane (4) and the position of the pressure maximum.

3) The ratio α ($= |p|/|p_0|$) of the sound pressure

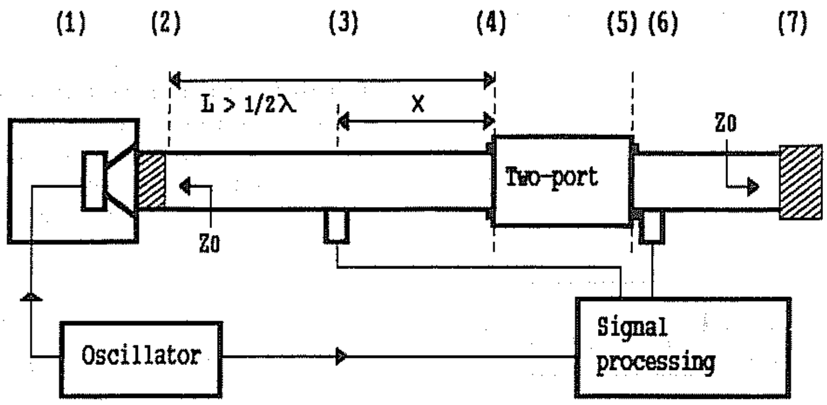


Fig. 4. Well-tested implementation of acoustic network analyzer. (1) loudspeaker; (2) absorbing material; (3) shifting microphone; (4) first reference plane; (5) second reference plane; (6) fixed microphone; (7) absorbing material.

measured with the fixed microphone (6) for the two-port connected between the two waveguides (p) and for both waveguides connected directly together (p_0).

4) The phase difference ϕ [$= \arg(p) - \arg(p_0)$], measured under the same conditions as described for α.

The complex parameters s_{11} and s_{21} can be reconstructed from these four real numbers as follows:

$$s_{11} = \frac{\mathrm{SWR} - 1}{\mathrm{SWR} + 1} \, e^{j4\pi x/\lambda}$$

$$s_{21} = \alpha \, e^{j\phi} \, .$$

The values of s_{22} and s_{12} can be found in the same way with the two-port measured in the reverse direction.

The acoustic network analyzer described here has been constructed and was used successfully for measurements on the feedback network of a thermoacoustic oscillator.

5 MATHEMATICS WITH ACOUSTIC S PARAMETERS

For simple acoustic systems, such as a cascade of segments, calculation of the dynamic properties of the overall system can be done by hand. Two examples will be worked out, 1) the complex relation between incident and reflected waves in case a two-port is terminated with a known acoustic reflection, and 2) the two-port S parameters of a cascade of separate two-ports. In both cases all S parameters of the relevant segments are assumed to be well known (by calculating and/or measurement).

5.1 Transformation of Acoustic Reflection

Assume an acoustic system that is terminated with a partially reflective element (Fig. 5). The complex reflection coefficient can be represented by a ratio number Γ_L at the termination point. At the output reference plane of the two-port a virtual reflection Γ_L' is observed, where the phase shift between Γ_L and Γ_L' of this reflection is $2\phi = 2(2\pi x/\lambda)$. The reflected waves propagate backward through the two-port. Now we observe a virtual reflection Γ_{in} at the input reference plane as if this reflection were generated by the input of the two-port.

Based on the definitions given, we found the following after some algebraic manipulations:

$$\psi_1^- = s_{11}\psi_1^+ + s_{12}\psi_2^-$$

$$\psi_2^+ = s_{21}\psi_1^+ + s_{22}\psi_2^- \, .$$

Letting $\Delta s = s_{11}s_{22} - s_{21}s_{12}$,

$$\psi_1^- = \frac{(s_{11}\psi_2^+ - \Delta s \psi_2^-)}{s_{21}}$$

$$\psi_1^+ = \frac{\psi_2^+ - s_{22}\psi_2^-}{s_{21}}$$

and hence,

$$\Gamma_L' = \frac{\psi_2^-}{\psi_2^+} = \Gamma_L \, e^{-2j\phi}$$

$$\Gamma_{in} = \frac{\psi_1^-}{\psi_1^+} = \frac{s_{11} - \Delta s \Gamma_L'}{1 - s_{22}\Gamma_L'} \, .$$

5.2 Cascading Acoustic Segments

When an acoustic system is formed by a chain of individual two-port networks, the overall network can also be represented by a single two-port (Fig. 6). Rearranging the two-port equations in another matrix format, in a way that the input waves become the dependent variables, the overall matrix can be evaluated by straightforward matrix multiplication:

$$\psi_1^- = s_{11a}\psi_1^+ + s_{12a}\psi_2^-$$

$$\psi_2^+ = s_{21a}\psi_1^+ + s_{22a}\psi_2^- \, .$$

Letting $\Delta s_a = s_{11a}s_{22a} - s_{21a}s_{12a}$,

$$\psi_1^+ = \frac{\psi_2^+ - s_{22a}\psi_2^-}{s_{21a}}$$

$$\psi_1^- = \frac{s_{11a}\psi_2^+ - \Delta s_a \psi_2^-}{s_{21a}}$$

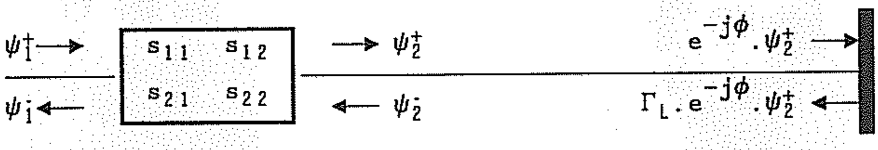


Fig. 5. Two-port terminated with known reflection.

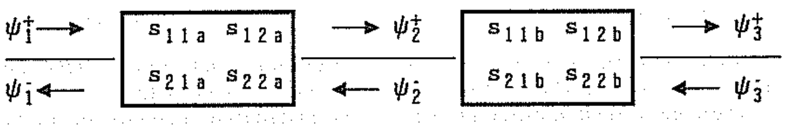


Fig. 6. Cascading two-ports.

and

$$\begin{bmatrix}\psi_1^+\\ \psi_1^-\end{bmatrix} = \frac{1}{s_{21a}}\begin{bmatrix}1 & -s_{22a}\\ s_{11a} & -\Delta s_a\end{bmatrix}\begin{bmatrix}\psi_2^+\\ \psi_2^-\end{bmatrix} .$$

Hence,

$$\begin{bmatrix}\psi_1^+\\ \psi_1^-\end{bmatrix} = \frac{1}{s_{21a}s_{21b}}\begin{bmatrix}1 & -s_{22a}\\ s_{11a} & -\Delta s_a\end{bmatrix} \cdot \begin{bmatrix}1 & -s_{22b}\\ s_{11b} & -\Delta s_b\end{bmatrix}\begin{bmatrix}\psi_3^+\\ \psi_3^-\end{bmatrix} .$$

Based on this application, the coefficients of the rearranged matrix are called transfer parameters. Calculating the S parameters of a cascade simply consists of transforming S parameters into T parameters, followed by matrix multiplication and completed by back transformation of the T parameters into S parameters. The relation between S and T parameters is

$$\begin{bmatrix}\psi_1^+\\ \psi_1^-\end{bmatrix} = \begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix}\begin{bmatrix}\psi_2^+\\ \psi_2^-\end{bmatrix} \rightarrow T = \frac{1}{s_{21}}\begin{bmatrix}1 & -s_{22}\\ s_{11} & -\Delta s\end{bmatrix}$$

$$\begin{bmatrix}\psi_1^-\\ \psi_2^+\end{bmatrix} = \begin{bmatrix}s_{11} & s_{12}\\ s_{21} & s_{22}\end{bmatrix}\begin{bmatrix}\psi_1^+\\ \psi_2^-\end{bmatrix} \rightarrow S = \frac{1}{t_{11}}\begin{bmatrix}t_{21} & \Delta t\\ 1 & -t_{12}\end{bmatrix}$$

with $\Delta t = t_{11}t_{22} - t_{21}t_{12}$.

6 CONCLUSION

Every segment of a linear acoustic two-port in which plane waves propagate can be represented by a linear N-port. This is a full characterization of the segment and includes delay time and phase effects. For this reason the properties of the overall system can be calculated even in case where there is no model of the individual two-ports available (black box). In contradiction to classical methods based on lumped circuit elements, a full N-port description can also be used for large segments with respect to the acoustic wavelength.

The representation of N-ports by S parameters is well suited for systems in which plane acoustic waves are propagating. These S parameters have physical significance and can therefore be measured directly.

The analysis of cascaded two-ports can be performed by manual calculations. For more complicated systems this analysis can be performed with commercially available simulation software developed for microwave circuits.

7 ACKNOWLEDGMENT

The method presented in this paper for describing acoustic networks was developed, and has been proven successful during the research on thermoacoustic energy-conversion systems within V.O.F. ASTER Thermoakoestische Systemen by C. M. de Blok and N. A. H. J van Rijt. The mathematical background and argumentation of the chosen parameterization were worked out by R. F. M. van den Brink, working at PTT-Research in the field of analog electronics and microwave technology.

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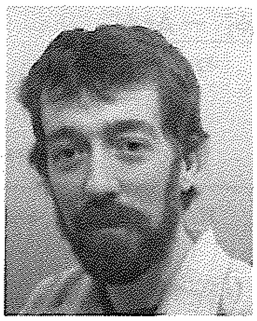

C. M. de Blok

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C. M. (Kees) de Blok was born 1954 July 6. After an electrotechnical education he joined PTT-Research in August of 1971. There he worked in different telecommunication areas, such as audio/video transmission, scribofonie, and digital filters, until 1978. He then became involved with the development of measuring and installation methods for the first direct detection optical fiber links in The Netherlands. In this field he

gained two patents: one for a directional coupler; another for a fiber alignment system. He was also coauthor of a publication on fiber installation.

Since 1984 he has worked on the development and realization of different coherent optical links and has been coauthor of a few conference proceedings on distribution and subscriber networks.

In addition to his work at PTT-Research, in 1985 he started research on thermoacoustic energy conversion. He is now working with others on modeling the complete thermoacoustic process.

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R. F. M. (Rob) van den Brink was born 1955 May 21. He graduated in 1984 from the Technical University Delft, in The Netherlands, with a degree in electrotechnical engineering. His graduation project concerned the development of feeds for microwave antennas. He joined the same university to work on microwave remote sensing experiments, but in 1985 joined PTT-Research in The Netherlands. Since then he has worked on fiber-optical transmission systems. He developed wideband optical receivers and transmitters operating up to a few GHz, and various new measurement and software design tools for opto-electronic circuits. His group designed various analog key circuits for coherent optical transmission systems, both for internal projects and for an ESPRIT (UCOL) project supported by the EEC.

Currently he is working for his thesis on a systematic design approach for wideband analog circuits, applied to high-performance optical receivers. His approach is extensively based on measurements of the key components in a circuit, postprocessing on the measured raw data, parameter extraction, and computer simulation.